MA2160
Test 1
Spring 2007

Name:_____

Non-calculator section: You may not use your calculator on this section. You must show enough work to justify all answers. Once you have handed in this section you may not have it back.

Page	Score	
1		/ 15
2		/ 8
3		/ 40
Non-calculator total		/ 63
Calculator total		/ 37
Final score		/ 100

1. Consider the vectors $\vec{u} = -\hat{\imath} + 2\hat{\jmath} + 7\hat{k}$, $\vec{v} = 2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ and $\vec{w} = 4\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$. Find

   (a) $\vec{u} \cdot \vec{v}$

-31 [5]

   (b) $\vec{v} \times \vec{w}$

$24\hat{\imath} - 16\hat{\jmath}$ [5]

   (c) A unit vector in the direction of $\vec{u}$.

$\frac{-1}{3\sqrt{6}}\hat{\imath} + \frac{2}{3\sqrt{6}}\hat{\jmath} + \frac{7}{3\sqrt{6}}\hat{k}$ [5]

2. Find an equation of the plane which is perpendicular to $4\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and contains the point (3,-6,2). [8]

$$4x + 2y - z = -2$$

3. Evaluate the following integrals.

(a) $\int 12y(y^2+6)^3\ dy$

$\frac{3(y^2+6)^4}{2}+c$ [8]

(b) $\int x\sqrt{x-1}\ dx$

$\frac{2(x-1)^{\frac{5}{2}}}{5}+\frac{2(x-1)^{\frac{3}{2}}}{3}+c$ [8]

(c) $\int \frac{1}{z^2+z-6}\ dz$

$-\frac{1}{5}\ln|z+3|+\frac{1}{5}\ln|z-2|+c$ [8]

(d) $\int_0^{\pi}\cos^2 x\ \sin x\ dx$

$\frac{2}{3}$ [8]

(e) $\int x\cos(2x)\ dx$

$\frac{x\sin(2x)}{2}+\frac{\cos(2x)}{4}+c$ [8]

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Calculator section: You may use your calculator on this section after turning in the non-calculator section. You must show enough work to justify all answers.

Page	Score	
1		/ 15
2		/ 10
3		/ 12
Calculator total		/ 37

1. Indicate whether the statement is true or false by putting a "T" or "F" in the blank following each statement. If the statement is false, explain why.

   (a) $\vec{v} \cdot \vec{w} = 0$ if and only if $\vec{v} = \vec{0}$ or $\vec{w} = \vec{0}$.
   $\vec{v}$ and $\vec{w}$ may be nonzero and still have a dot product of 0.

   F [3]

   (b) The zero vector is orthogonal to any other vector.

   T [3]

   (c) The dot product of a vector with itself is its magnitude.
   $\vec{u} \cdot \vec{u} = ||\vec{u}||||\vec{u}|| \cos 0 = ||\vec{u}||^2$

   F [3]

   (d) Any vector normal to a surface has length one.
   A normal vector may have any length as long as it is perpendicular to a surface.

   F [3]

   (e) If two planes are perpendicular, so are their normal vectors.

   T [3]

2. There are three hands on a clock, the hour hand, the minute hand and the second hand. View the second and hour hands of a clock as vectors $\vec{s}$ and $\vec{h}$ in a horizontal plane in 3-space. Suppose $\vec{h}$ points at 12 o'clock. At what time(s) during the next minute does $\vec{s} \times \vec{h}$ have largest magnitude? Explain. [10]

   The magnitude of $\vec{s} \times \vec{h}$ is equal to the area of a parallelogram formed by the two vectors. Therefore, the area will be largest when the two vectors are at a 90° angle. This happens after 15 seconds and again after 45 seconds. Therefore, the times are 12:00:15 and 12:00:45.

3. Last Monday at an office a typist was hired to work from 8am to 12 noon. His typing speed decreased between 8am and his 10am cup of coffee, and increased again afterwards, between 10am and noon. His instantaneous speed (measured in characters per second) was measured each hour and the results are given below:

Time	8am	9	10	11	12
Speed	6	4	1	3	5

(a) Use the trapezoidal approximation to make an estimate for the total numbers of characters typed between 8am and 12 noon.

48600 characters [8]

(b) A good typist types at an average speed of at least four characters per second. Use your answer above to decide if a good typist was hired. Explain your reasoning. [4]

The typist types at an average speed of 3.375 characters per second. That is less than 4 characters per second. Thus, the typist is not good.